

Dear Calculus BC Student:

I am very excited to have the opportunity to work with you as we explore Calculus next year. In order to make the most of our time together, it is critical that we come back to school ready to learn new material. In order to help you achieve that end, I have attached a summer homework assignment.

The packet contains worksheets on topics that you have already learned, but that need to be fresh as we move forward. It is, therefore, recommended that you do this towards the end of August, shortly before returning to school. Yes, I am asking you to procrastinate this until the end of summer (but make sure it is done before school starts as it will account for a large portion of your starting grade).

Have a great summer, and see you this fall!

Mr. Matheny



## Simplifying Rational Expressions

Simplify each expression.

1)  $-\frac{36x^3}{42x^2}$

2)  $\frac{16r^2}{16r^3}$

3)  $\frac{16p^2}{28p}$

4)  $\frac{32n^2}{24n}$

5)  $-\frac{70n^2}{28n}$

6)  $\frac{15n}{30n^3}$

7)  $\frac{2r-4}{r-2}$

8)  $\frac{45}{10a-10}$

9)  $\frac{x-4}{3x^2-12x}$

10)  $\frac{15a-3}{24}$

11)  $\frac{v-5}{v^2-10v+25}$

12)  $\frac{x+6}{x^2+5x-6}$

$$13) \frac{27}{27x + 18}$$

$$14) \frac{v^2 - 7v - 30}{v^2 - 5v - 24}$$

$$15) \frac{x^2 + 8x + 12}{x^2 + 3x - 18}$$

$$16) \frac{x^2 - 11x + 18}{x^2 + 2x - 8}$$

$$17) \frac{b^2 + 3b - 28}{b^2 - 49}$$

$$18) \frac{v^2 - 3v - 40}{v^2 - 11v + 24}$$

$$19) \frac{4n - 4}{6n - 20}$$

$$20) \frac{v^2 - 5v - 14}{v^2 + 4v + 4}$$

$$21) \frac{6v^3 + 42v^2}{2v^2 + 26v + 84}$$

$$22) \frac{x^3 - x^2 - 42x}{2x^2 - 20x + 42}$$

$$23) \frac{2v^2 + 10v - 48}{8v + 64}$$

$$24) \frac{9x^2 + 81x}{x^3 + 8x^2 - 9x}$$

$$25) \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

$$26) \frac{3r^2 - 39r + 90}{r^2 - 3r - 70}$$

## Properties of Logarithms

**Expand each logarithm.**

1)  $\log \frac{2}{3}$

2)  $\log (3 \cdot 11)$

3)  $\log (6 \cdot 7)$

4)  $\log (5 \cdot 11)$

5)  $\log (7 \cdot 8)$

6)  $\log \frac{12}{11}$

7)  $\log \sqrt[3]{x}$

8)  $\log \sqrt{x}$

9)  $\log (a \cdot b)$

10)  $\log x^5$

11)  $\log (u \cdot v)^6$

12)  $\log (ab^5)$

**Condense each expression to a single logarithm.**

13)  $\log 6 - \log 5$

14)  $\log 12 + \log 5$

15)  $\log 6 + \log 7$

16)  $\log 12 - \log 11$

17)  $3\log x$

18)  $6\log a$

19)  $\log a - \log b$

20)  $\frac{\log x}{2}$

21)  $\log x + 5\log y$

22)  $6\log u - 6\log v$

23)  $4\log x + 4\log y$

24)  $\log u + \log v + \log w$

**Critical thinking questions:**

25)  $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

26)  $\log x \cdot \log 2$

Prove each identity:

$$1. \quad \sec x - \tan x \sin x = \frac{1}{\sec x}$$

$$2. \quad \frac{1 + \cos x}{\sin x} = \csc x + \cot x$$

$$3. \quad \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$4. \quad \frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

$$5. \quad \cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

$$6. \quad \csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$$

$$7. \quad \frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

$$8. \quad \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$9. (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$$

$$10. (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$$

$$11. \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1 - \cot\theta}{1 + \cot\theta}$$

$$12. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$13. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$14. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$15. \frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$$

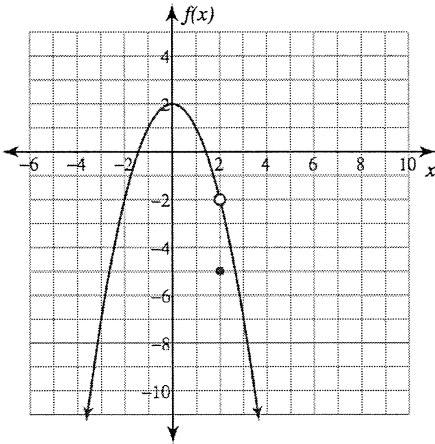
$$16. \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$



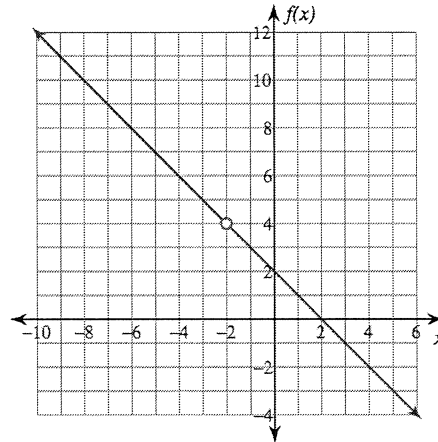
## Evaluating Limits

Evaluate each limit.

$$1) \lim_{x \rightarrow 2} f(x), f(x) = \begin{cases} -x^2 + 2, & x \neq 2 \\ -5, & x = 2 \end{cases}$$

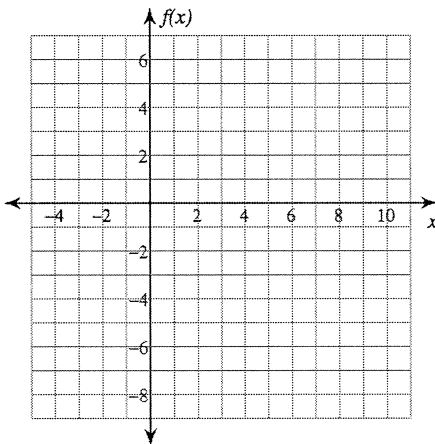


$$2) \lim_{x \rightarrow -2} -\frac{x^2 - 4}{x + 2}$$

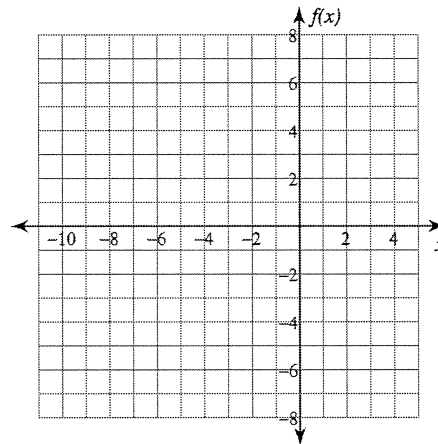


Evaluate each limit. You may use the provided graph to sketch the function.

$$3) \lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3}$$



$$4) \lim_{x \rightarrow -3} \frac{x + 3}{x^2 + 2x - 3}$$



Evaluate each limit.

$$5) \lim_{x \rightarrow 0} f(x), f(x) = \begin{cases} x + 1, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

$$6) \lim_{x \rightarrow 3} f(x), f(x) = \begin{cases} 2 + \frac{x}{2}, & x \neq 3 \\ 2, & x = 3 \end{cases}$$

7)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

8)  $\lim_{x \rightarrow 5} \frac{x^2 - 5x}{x - 5}$

9)  $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

10)  $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x + 5}$

11)  $\lim_{x \rightarrow 0} \frac{\frac{1}{-4 + x} + \frac{1}{4}}{x}$

12)  $\lim_{x \rightarrow -3} \frac{x}{\frac{1}{3 + x} - \frac{1}{3}}$

13)  $\lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x + 4} - 3}$

14)  $\lim_{x \rightarrow 3} \frac{\sqrt{x + 6} - 3}{x - 3}$

**Critical thinking questions:**

15) Give an example of a limit of a rational function where the limit at -1 exists, but the rational function is undefined at -1.

16) Give two values of  $a$  where the limit cannot be solved using direct evaluation. Give one value of  $a$  where the limit can be solved using direct evaluation.

$$\lim_{x \rightarrow a} \frac{x}{\frac{1}{-2 + x} + \frac{1}{2}}$$

## Differentiation - Chain Rule

**Differentiate each function with respect to  $x$ .**

1)  $y = (x^3 + 3)^5$

2)  $y = (-3x^5 + 1)^3$

3)  $y = (-5x^3 - 3)^3$

4)  $y = (5x^2 + 3)^4$

5)  $f(x) = \sqrt[4]{-3x^4 - 2}$

6)  $f(x) = \sqrt{-2x^2 + 1}$

7)  $f(x) = \sqrt[3]{-2x^4 + 5}$

8)  $y = (-x^4 - 3)^{-2}$

$$9) y = (3x^3 + 1)(-4x^2 - 3)^4$$

$$10) y = \frac{(x^3 + 4)^5}{3x^4 - 2}$$

$$11) y = ((x + 5)^5 - 1)^4$$

$$12) y = (5x^3 - 3)^5 \sqrt[4]{-4x^5 - 3}$$

**Critical thinking question:**

- 13) Give a function that requires three applications of the chain rule to differentiate. Then differentiate the function.

## Differentiation Rules, with Tables

For each problem, you are given a table containing some values of differentiable functions  $f(x)$ ,  $g(x)$  and their derivatives. Use the table data and the rules of differentiation to solve each problem.

1) 

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	1	2
2	3	0	3	0
3	2	-1	1	-2

Given  $h(x) = f(x) + g(x)$ , find  $h'(1)$

2) 

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	1
2	1	$\frac{1}{2}$	2	1
3	3	2	3	1

Given  $h(x) = f(x) - g(x)$ , find  $h'(2)$

3) 

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	-2

Given  $h(x) = f(x) \cdot g(x)$ , find  $h'(3)$

4) 

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	2	-1
2	2	-1	1	0
3	1	-1	2	1

Given  $h(x) = \frac{f(x)}{g(x)}$ , find  $h'(3)$

5)

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	-1	4	1
4	2	-1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	-2

Part 1) Given  $h_1(x) = f(x) + g(x)$ , find  $h_1'(2)$

Part 2) Given  $h_2(x) = f(x) - g(x)$ , find  $h_2'(3)$

Part 3) Given  $h_3(x) = f(x) \cdot g(x)$ , find  $h_3'(4)$

Part 4) Given  $h_4(x) = \frac{f(x)}{g(x)}$ , find  $h_4'(2)$

Part 5) Given  $h_5(x) = (f(x))^2$ , find  $h_5'(2)$

Part 6) Given  $h_6(x) = f(g(x))$ , find  $h_6'(6)$